Asphalt Concrete Material Characterization Using a Music-Inspired Optimization Algorithm

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ABSTRACT

This research documents the procedure for determining viscoelastic properties using a harmony search (HS) algorithm that employs a heuristic algorithm based on an analogy with natural phenomena. To determine the viscoelastic material parameters, the steps involved in conducting the interconversion between frequency-domain and time-domain functions are outlined, based on the presmoothing of raw data using the HS algorithm. Thus, a Prony series representation of the fitted data can be obtained that includes the determination of the Prony series coefficients.

1. INTRODUCTION

Linear viscoelastic (LVE) materials are rheological materials that exhibit time-temperature rate-of-loading dependence. When their response is not only a function of the current input, but also of the current and past input history, the characterization of the viscoelastic response can be expressed using the convolution (hereditary) integral. A general overview of time-dependent material properties has been presented [1]. Additionally, a detailed description of the physical response of LVE materials has been explained [2], based on ramp tests to determine the relaxation modulus, which is a time-domain LVE response function.

Hot mix asphalt (HMA) concretes used in this study are composite materials consisting of aggregates and asphalt binder. Their behavior is characterized by the interaction between these two components and the LVE behavior of the HMA concretes, which depends on temperature, loading frequency, and strain magnitude. Studying the behavior of the HMA material requires modeling the LVE behavior through a dynamic modulus test (DMT) conducted in stress-control within the LVE range. This test is run on all previously untested specimens to obtain a LVE fingerprint (linear viscoelastic properties characteristic of HMA specimen) and to determine the shift factors for the undamaged state after constructing dynamic modulus and phase angle mastercurves. Sinusoidal loading in tension and compression sufficient to produce total strain amplitude of about 70 micro-strains was applied at different frequencies. Based on earlier work [3], the 70 micro-strain limit was found not to cause significant damage to the specimen. For mastercurve construction, several replicates are tested at four temperatures: –10, 5, 25 and 40°C.

Several methods have been proposed to convert the dynamic modulus, a LVE response function in the frequency domain, to the corresponding relaxation modulus in the time domain. For a time-domain LVE function, the Prony series is a popular representation, mainly because of its ability to describe a wide range of viscoelastic response and its relatively simple and rugged computational efficiency associated with its exponential basis functions. In this study, an approach is proposed to overcome the problem – namely, oscillations in the fitted curve – associated with the Prony series fitting. The experimental source data are smoothed through a defined log-sigmoidal function using the heuristic optimization technique of the harmony search (HS) algorithm. The coefficients of the log-sigmoidal function can be determined when a defined error norm converges into the minimum point. Thus, this procedure is required to represent the oscillated broadband data using the smoothed log-sigmoidal function. The fitted log-sigmoidal function is used to obtain the time-domain Prony series. This approach has proved very effective and stable in fitting a Prony series with positive coefficients to LVE response function data, as illustrated in an example using experimental data from a dynamic modulus test on HMA concrete.

2. HARMONY SEARCH ALGORITHM

Many engineering optimization problems are very complex in nature and quite difficult to be solved using gradient-based search algorithms. If there is more than one local optimum in the problem, the result may depend on the selection of an initial point, and the obtained optimal solution may not necessarily be the global optimum. To determine the LVE and damage properties of HMA concrete, the HS algorithm based on analogies with natural or artificial phenomena was used in this study.

The HS algorithm conceptualizes a behavioral phenomenon of musicians in the improvisation process, where each musician continues to experiment and improve his or her contribution in order to search for a better state of harmony [4, 5].

3. DETERMINATION OF LVE MATERIAL PARAMETERS

In order to get the experimental data to be fitted in this study, a DMT is performed by controlling a micro-strain level of 70 that is targeted as the limit for the LVE. The loading is applied until steady-state response is achieved, at which point several cycles of data are collected. After each frequency, a five-minute rest period is allowed for specimen recovery before the next loading block is applied. The frequencies are applied from the fastest to the slowest ranging from 1 to 20 Hz.

From the DMT, the complex modulus, E^*, which includes the dynamic modulus ($|E^*|$) and the phase angle (ϕ), can be determined. The complex modulus can also be viewed as a composition of storage (E') and loss modulus (E'') as follows:

$$E^* = E' + iE'' \qquad (1)$$

where i is the $\sqrt{-1}$. The dynamic modulus is the amplitude of the complex modulus and is defined as:

$$|E^*| = \sqrt{(E')^2 + (E'')^2} \; . \tag{2}$$

The values of the storage and loss modulus are related to the dynamic modulus and phase angle as follows:

$$E' = |E^*|\cos\phi \quad \text{and} \quad E'' = |E^*|\sin\phi \, . \tag{3}$$

As the material becomes more viscous, the phase angle increases and the loss component of the complex modulus increases. Conversely, a decreasing phase angle indicates more elastic behavior and a larger contribution from the storage modulus. The dynamic modulus at each frequency is calculated by dividing the steady state stress amplitude, σ_{amp}, by the strain amplitude, ε_{amp}:

$$|E^*| = \frac{\sigma_{amp}}{\varepsilon_{amp}} \, . \tag{4}$$

The phase angle, ϕ, is associated with the time lag, Δt, between the strain input and stress response at the corresponding frequency, f:

$$\phi = 2\pi f \Delta t \, . \tag{5}$$

In order to determine the storage modulus prior to the conversion (i.e., from frequency domain to time domain) using a Prony series function, the discrete raw data need to be fitted using a continuous function. Thus, the quality of raw data can be significantly improved using a defined log-sigmoidal function that describes the full viscoelastic range of the HMA, from the glassy state to the low frequency plateau. The log-sigmoidal function, $f(\omega)$, is defined as

$$f(\omega) = a_1 + \frac{a_2}{\left\{a_3 + \dfrac{a_4}{\exp\left[a_5 + a_6 \log_{10}(\omega)\right]}\right\}} \tag{6}$$

where $a_{1,2,\ldots,6}$ are the coefficients determined by the HS algorithm, and ω is the angular frequency. For example, the optimizing solution between the storage modulus data and the log-sigmoidal function that is used to determine the coefficients of Equation (6) can be expressed by the following Equation (7):

$$\text{MINIMIZE} \quad g(\omega_i) = \sum_{i=1}^{N} \left| \log_{10}\left[E'(\omega_i)\right] - f(\omega_i) \right| , \tag{7}$$

where $g(\omega_i)$ is the objective function (e.g., error norm); $E'(\omega_i)$ is the storage modulus; subscript i denotes the individually selected angular frequency; N is the total number of selected angular

frequencies; and the vertical bar indicates the absolute value.

In order to smooth the discrete raw data that need to be fitted using the defined log-sigmoidal function of Equation (6), the HS algorithm can be used based on minimizing the objective function of Equation (7). Figure 1 shows the error norm of the objective function defined by Equation (7) in terms of the number of iterations. Based on Figure 1, it is noted that the best solution converges as the number of iterations increases. Finally, Figure 2 shows the log-sigmoidal function smoothly fitted with the raw data.

At the low frequencies shown in Figure 2b, some minor loss of information may be resulted due to some local irregularities of the scattered data. However, the fitting function provides a smooth representation over all frequencies. Thus, the time-domain Prony-series representation can be determined using the log-sigmoidal function of Equation (6) as follows:

$$E(t) = E_\infty + \sum_{m=1}^{M} E_m \exp(-t/\rho_m) \tag{8}$$

where E_∞, ρ_m, and E_m are the infinite relaxation modulus, relaxation time, and Prony coefficients, respectively.

Considering the case of an infinite number of Maxwell components with continuously distributed relaxation times, and neglecting the infinite relaxation modulus, E_∞, based on the continuous spectrum method, the relaxation modulus can be defined by

$$E(t) = \int_{-\infty}^{\infty} L(\rho) \exp(-t/\rho) d\rho\ , \tag{9}$$

where $L(\rho)$ represents a continuous distribution of the relaxation modulus, and is defined by

$$L(\rho) = \lim_{k\to\infty} \frac{(-k\rho)^k}{(k-1)!} E^{(k)}(k\rho)\ . \tag{10}$$

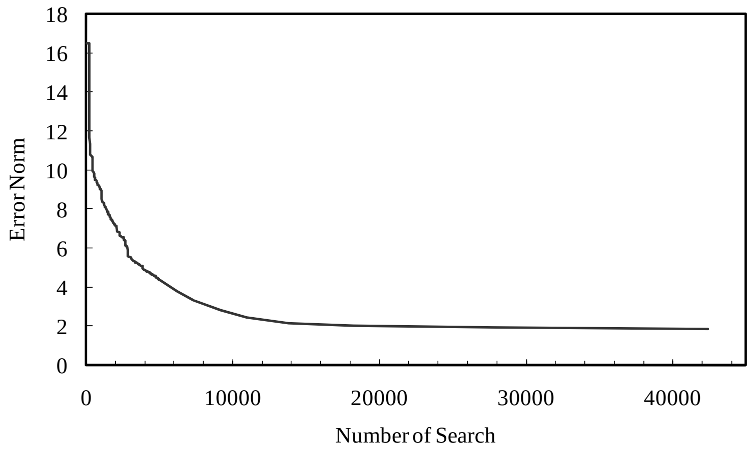
In order to determine the discrete Prony series coefficients, E_m, in Equation (8) from the continuous spectrum equation of Equation (9), the continuous spectrum can be approximated by subdividing $\ln\rho$ into time intervals $\Delta(\ln\rho_m) = \ln 10 \Delta(\log_{10}\rho_m)$, as follows:

$$E(t) = \int_{-\infty}^{\infty} L(\rho) \exp(-t/\rho) d\ln\rho \approx \sum_{m=1}^{M} L(\rho_m) \exp(-t/\rho_m) \Delta(\ln\rho_m)\ . \tag{11}$$

The Prony coefficients for the chosen relaxation time can be determined by

$$E_m = L(\rho_m) \ln 10 \Delta(\log_{10}\rho_m)\ . \tag{12}$$

(a)



(b)

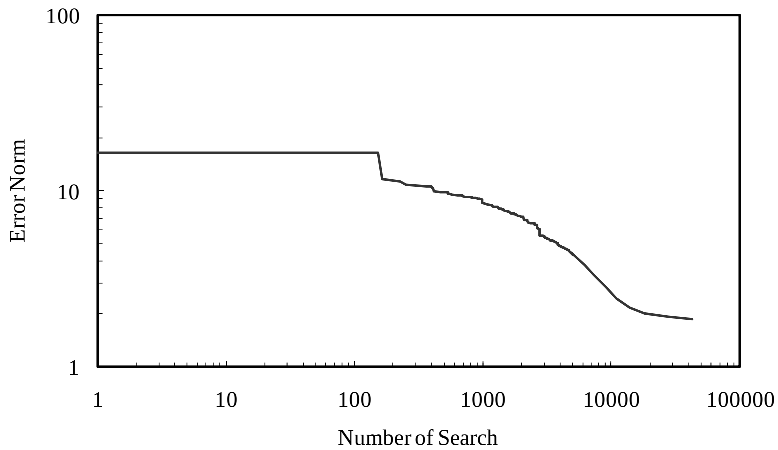
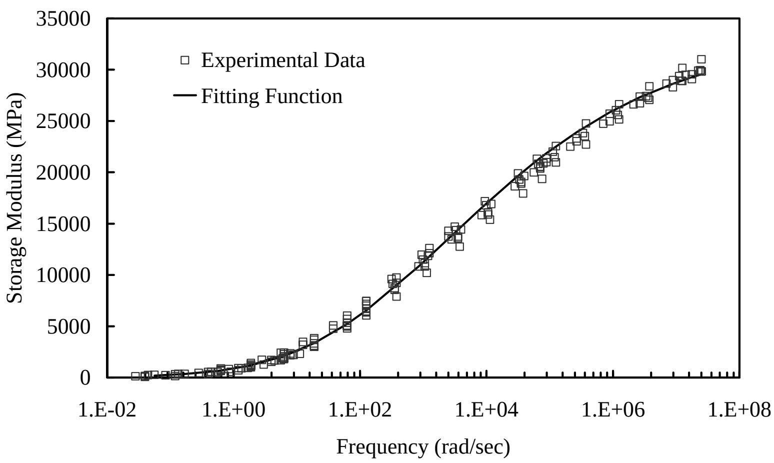


Figure 1. Convergence of the Objective Function during the Number of Searches: (a) Semi-Log Scale; (b) Log-Log Scale.

(a)



(b)

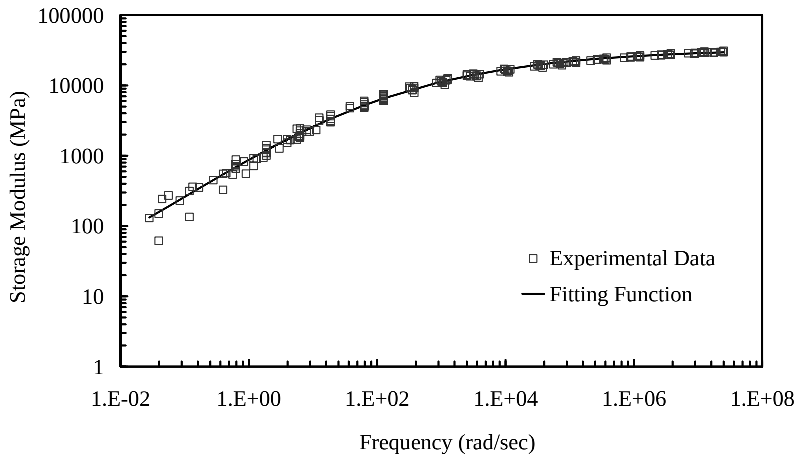


Figure 2. Fitting the Experimental Data of the Unmodified HMA to the Log-Sigmoidal Function: (a) Semi-Log Scale; (b) Log-Log Scale.

4. CONCLUSIONS

In this research, the HS algorithm has been implemented in the context of determining viscoelastic properties. An interconversion method between time- and frequency- domain LVE responses for HMA concrete is presented based on a methodology of the HS algorithm to pre-smooth the experimental data using the log-sigmoidal function before fitting to a Prony series. Through the application to the prediction of the linear viscoelastic behavior of HMA concrete, the time-domain Prony-series representation can be obtained from the frequency-domain experimental data.

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